Review of Algebra

8/23/16

Real Numbers and Mathematical Operations

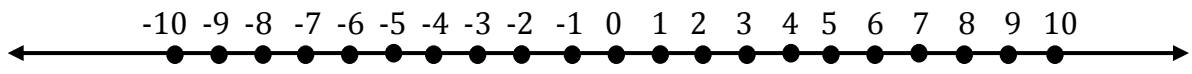
1. **Sets of Numbers**
   - *Natural numbers or Counting Numbers:* (1, 2, 3, 4, 5…)
   - *Whole Numbers:* {0, 1, 2, 3, 4, 5…}
     - Whole numbers include all natural numbers
   - *Integers:* {…, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5…}
     - Integers include negative numbers, whole numbers and natural numbers
   - *Rational Numbers:* the set of all numbers that can be expressed as a ratio of two integers where 0 is not in the denominator
     - Ratio is a comparison of numbers
     - Every integer is a rational number

2. ***Examples of Rational Numbers***
   - ½, -2/7
   - ½ = 0.5 → 2÷1 = 0.5 – This is a *terminating decimal* (all terminating decimals are rational numbers)
   - 0.73 = 73/100 This is a *terminating decimal*
   - $1\frac{1}{2} = \frac{2(1)+1}{2} = \frac{3}{2}$

- 1/3 = 0.3333… = 0.3 – this is a repeating decimal
- 6/3 = 2 because 3x2 = 6
- 7/7 = 1 because 7x1 = 7
- 5/0 = undefined
- 6 = 6/1 or 12/2 or 18/3

3. ***Irrational numbers:*** all real numbers that are not rational numbers (the can't be expressed as the ratio of two integers)
   a. Examples: $\sqrt{2}$, $\sqrt{5}$, π, $\sqrt[3]{6}$…
   b. Cool fact: irrational numbers have a non-terminating and non-repeating decimal property expansion
   c. $\pi = 3.14159265$

4. ***The Real Number Line****:* contains all the real numbers as points—the more to the right a number is, the greater the number



$\sqrt{9} < \sqrt{13} < \sqrt{16}$

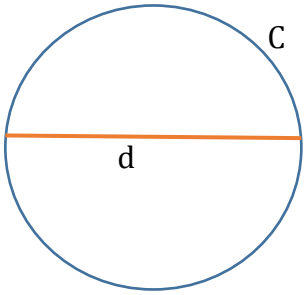
$3 < \sqrt{13} < 4$

< - Less than

> - Greater than

= - Equal to

$\sqrt{\ }$ – Square root



$\frac{c}{d} = \pi$

Need to Memorize!

$\sqrt{0} = 0$	$\sqrt{25} = 5$
$\sqrt{1} = 1$	$\sqrt{36} = 6$
$\sqrt{4} = 2$	$\sqrt{49} = 7$
$\sqrt{9} = 3$	$\sqrt{64} = 8$
$\sqrt{16} = 4$	$\sqrt{81} = 9$

$\sqrt{100} = 10$

Operations with Real Numbers

- **Absolute Value:** The absolute value of a number represents the distance on the number line between that number and 0
  - Absolute value bars: $|x|$
  - **Examples:**
    - $|3| = \mathbf{3}$
    - $|-(-8)| = |8| = \mathbf{8}$
    - $-|-4| = \mathbf{-4}$

- **Additive Opposites:** 2 numbers that are the same distance from 0 but on opposite sides of 0 on the number line are said to be additive inverse (or opposites) of each other. They add up to 0
  - Examples:
    - 10, -10
    - 0.34, -0.34
    - $-\frac{1}{2}, \frac{1}{2}$

- **Addition of Real Numbers**
  - Rule 1: If all numbers are positive, then add as usual, the answer is positive
  - Rule 2: If all numbers are negative then add as usual, the answer is negative
  - Rule 3: If one number is positive and other is negative then:
    - Find the absolute value of both numbers
    - Find the difference between the absolute values
    - Give the sign of the original number with the larger absolute value
  - Examples:
    - $5+2 = \mathbf{7}$
    - $-5+2 = \mathbf{-3}$
    - $-5-2 = -5 + (-2) = \mathbf{-7}$
    - $5-2 = 5+(-2) = \mathbf{3}$
    - $\frac{-7}{8} + \frac{3}{4} =$ You need the *same denominator* to add fractions
      - $\frac{-7}{8} + \frac{3\ x\ 2}{4\ x\ 2} = \frac{-7+6}{8} = \mathbf{\frac{-1}{8}}$
    - $-\frac{3}{5} - \frac{4}{3} = -\frac{3\ x\ 3=9}{5\ x\ 3=15} - \frac{4\ x\ 5=20}{3\ x\ 5=15} = \frac{-9-20}{15} = \mathbf{\frac{-29}{15}}$

Multiplication & Division of Real Numbers

$+ \times + = +$

$- \times - = +$

$+ \times - = -$

$- \times + = -$

Same for division

- $-\mathbf{1}\frac{\mathbf{1}}{\mathbf{3}} - \frac{\mathbf{7}}{\mathbf{8}} = -\frac{3(1)+1}{3} - \frac{7}{8} = -\frac{4x8}{3x8} - \frac{7x3}{8x3} = -\frac{32}{24} - \frac{21}{24} = \frac{-32-21}{24} = \frac{-53}{24}$
- 4.5
  x 2
  9.0

Multiplication Examples:

- $2(4) = 8$ $\quad 0(-5) = 0$
- $-2(-4) = 8$ $\quad -\frac{1}{2}\left(\frac{1}{4}\right) = -\frac{1}{8}$
- $2(-4) = -8$
- $-2(4) = -8$

Division Examples:

- $-\frac{1}{2} \div \left(-\frac{1}{2}\right) =$
- $-8 \div 2 = -4$
- $-8 \div (-2) = 4$
- $-\frac{1}{2} \div \left(-\frac{1}{2}\right) = 1$
- $0 \div 5 = 0$
- $6 \div 0 =$ Undefined

Reciprocal or Multiplicative Inverse: two numbers are reciprocals (or multiplicative inverse) of each other if their product equals 1.

- $\frac{3}{5}, \frac{5}{3}$
- $-\frac{2}{3}, -\frac{3}{2}$
- $\frac{6}{1}, \frac{1}{6}$
- $-\frac{1}{2} \div \left(-\frac{1}{2}\right) = -\frac{1}{2}x\left(-\frac{2}{1}\right) = \frac{1x2}{2x1} = \frac{2}{2} = 1$