

FIGURE 7-4

The remainder of this chapter discusses probability theory. The topics discussed in this section will be helpful in the discussion of probability and understanding some of the more difficult concepts in probability theory such as computing the probability of X "or" Y occurring (an example of union of possible outcomes) and the probability of X "and" Y occurring (an example of the intersection of possible outcomes).

Practice Set 7-1

Let set $A = \{1, 2, 3, 4, 5\}$ and let set $B = \{x \mid x \text{ is a natural number greater than } 4\}$.

Determine whether each of the following is true or false. (problems 1 to 12)

- 1. $2 \in A$
- 2. $10 \in B$
- 3. $8 \in A$
- 4. $6 \notin A$
- 5. $4 \in B$
- 6. $3 \notin A$
- 7. $B = \{3.5, 4, 4.5, 5, 5.5, 6, 6.5, \dots\}$
- 8. $A \cup \emptyset = \emptyset$
- 9. $\{\emptyset\} = \emptyset$
- 10. $A \cap B = \emptyset$
- 11. $\{2, 3\} \subseteq A$
- 12. Set A is a finite set.
- 13. Write the set $A = \{1, 2, 3, 4, 5\}$ in set builder notation.
- 14. Write, in set notation, the set of natural numbers between 2 and 13.
- 15. Write, in set notation, the set of even natural numbers less than 50.
- 16. Write the set {January, February, March, ..., December} in set-builder notation.

Label each of the following as a finite set or an infinite set. (problems 17 to 20)

- 17. $\{1, 2, 3\}$
- 18. $\{x \mid x \text{ is an integer such that } 5 \leq x \leq 11\}$

- 19. $\{x \mid x \text{ is a number such that } 0 \leq x \leq 8\}$
- 20. $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Let set $A = \{1, 2, 3, 4, 5, 6\}$, set $B = \{2, 4, 6, 8\}$, set $C = \{x \mid x \text{ is an even natural number}\}$, and set $D = \{x \mid x \text{ is an odd natural number}\}$. (problems 21 to 24)

- 21. Write $A \cup B$
- 22. Write $A \cap B$
- 23. Write $B \cap D$
- 24. Which set is a subset of one of the others?

Given $M = \{1, 2, 3, 4, 5\}$; $N = \{1, 4, 7\}$; and $P = \{2, 3, 5, 6, 8\}$. Perform each of the following operations and draw a Venn diagram to illustrate your answers. (problems 25 to 30)

- 25. $M \cap N$
- 26. $P \cap N$
- 27. $M \cup P$
- 28. $P \cup N$
- 29. $M \cap P$
- 30. $M \cup N$

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; $A = \{1, 2, 3, 4, 5\}$; $B = \{1, 2, 5, 7\}$; and $C = \{3, 5, 7, 9\}$. (problems 31 to 40)

- 31. $A \cup B$
- 32. $A \cup C$
- 33. $B \cup C$
- 34. $A \cap B$
- 35. $A \cap C$
- 36. $B \cap C$
- 37. $A \cap \emptyset$
- 38. $A \cup \emptyset$
- 39. $A \cap (B \cap C)$
- 40. $(B \cap C) \cap A$

- (c) What is the probability that you will get poor service, given that you chose to have your repairs done at the car dealer's shop?
There were 29 people who had their car repaired at the car dealership, and 9 of them reported poor service.

$$P(\text{poor service} | \text{car dealer's shop}) = \frac{9 \text{ poor service}}{29 \text{ responses about car dealer's shop}} = 31\%$$

Practice Set 7-2

Answer problems 1 to 4 in complete sentences.

1. What is an experiment? What are outcomes of an experiment? What is an event?
2. Explain how you would find the empirical probability of rolling an odd number on a die. What do you believe is the empirical probability of rolling an odd number? Find the empirical probability of rolling an odd number by rolling a die 40 times.
3. Explain in your own words the law of large numbers.
4. To determine life insurance premiums, life insurance companies must compute the probable date of death. On the basis of research done by the company, Mr. Timmons, age 53, is expected to live another 23.25 years. Does this mean that Mr. Timmons will live until he is 76.25 years old? If not, what does it mean?

Classify the following events as independent or dependent events.

5. Randomly selecting a voter who is a Democrat and randomly selecting a voter who is a Republican.
6. Winning the lottery in Virginia and winning the lottery in North Carolina.
7. In a club meeting of 20 students, 12 boys and 8 girls, the probability of a boy being selected president and then the probability of another boy being selected vice president.
8. A person lies, and a person fails a lie detector test.

Calculate each probability in percent form rounding to the nearest tenth.

9. In a sample of 60,000 births, 155 were found to have Down syndrome. Find the empirical probability that Mrs. Cosgrove's first child will be born with this syndrome.
10. A certain community recorded 4000 births last year, of which 2350 were female. What is the empirical probability that the next child born in the community will be a female? What is the empirical probability that the next child born will be a male?
11. The McCooks are participating in Project FeederWatch sponsored by the Cornell Lab of Ornithology. The last 30 birds at the McCooks' bird feeder were 20 robins, 5 cardinals, 3 blue jays, and 2 doves. Find the empirical probability that the next bird to feed from the feeder will be a cardinal. Find the empirical probability that the next bird will be a robin.
12. Some years ago, a survey was done of men aged 35 and women aged 35 who were married to each other. A total of 81,800 couples was surveyed. Thirty years later, only 64,528 men and 71,862 women were still living. Based on these limited data:
 - (a) What is the probability that a male aged 35 will live another 30 years?
 - (b) What is the probability that a female aged 35 will live another 30 years?
13. Of the last 70 people who went to the cash register at Wal-Mart, 12 had blond hair, 20 had

black hair, 33 had brown hair, and 5 had red hair. Determine the empirical probability that the next person to come to the cash register will have red hair.

14. Of the last 70 people who went to the cash register at Wal-Mart, 12 had blond hair, 20 had black hair, 33 had brown hair, and 5 had red hair. Determine the empirical probability that the next person to come to the cash register will have blond hair.
15. Mr. Harclerode's grade distribution over the past 3 years in Algebra I is shown in the following chart.

Grade	Number of Students
A	41
B	183
C	265
D	96
F	85
Incomplete	2

If Sharon Odell plans on taking Algebra I with Mr. Harclerode, determine the empirical probability that she will receive a grade of A.

16. If Lafawn Oliver plans on taking Algebra I with Mr. Harclerode, determine the empirical probability that she will receive a grade of C. (Use the table in the previous problem.)
17. Six friends chartered a deep-sea fishing boat for a day's fishing trip. They caught a total of 72 fish. The following chart provides information about the type and number of fish caught.

Fish	Number Caught
Shark	8
Flounder	35
Kingfish	10
Grouper	19

Determine the empirical probability that the next fish caught will be a shark.

18. Six friends chartered a deep-sea fishing boat for a day's fishing trip. They caught a total of 72 fish. Determine the empirical probability that the next fish caught will be a flounder. (Use the chart in the previous problem.)

When a new medication is being studied by a drug company, test groups are asked to report side effects and probabilities of those side effects are calculated based on the results from the test group. A new antidepressant is being tested using a group of 120 people. The number of people reporting specific side effects during the study are listed in the table below. Use this table for problems 19 to 22. Express all probabilities as percents.

Side Effects	Number of Participants Reporting a Side Effect
Headache	27
Nausea	24
Insomnia	15
Fatigue	10
Dizziness	6

19. Using the numbers reported in the table, calculate the probability of a person experiencing insomnia while taking this medication.
20. What is the probability of someone experiencing nausea while taking this medication?
21. What is the probability of someone experiencing dizziness while taking this antidepressant? Based on this probability, if 25,000 people take this medication, how many would you expect to experience dizziness as a side effect?
22. What is the probability of someone experiencing headaches while taking this antidepressant? If 500,000 people take this medication, how many would you expect to experience headaches as a side effect?

Body mass index (BMI) is now used by many people to determine if a person is overweight. Using this scale, a BMI ≥ 25.0 classifies an individual as overweight. There were 55 new members at a women's fitness club this month. Each woman had her BMI calculated when she joined. The results are posted in the table below. Use this table to answer problems 23 to 26.

Body Mass Index of Women	Frequency
15.0–19.9	9
20.0–24.9	12
25.0–29.9	25
30.0–34.9	6
35.0–39.9	2
40.0–44.9	1

23. If c
the
wh
24. If
as
th
yc
pi
25. If
fr
ri
a
26. I
f
s
f
c

The
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beca
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Use

O+
O-
A+
A-
B+
B-
A1
A1
2

2

Practice Set 7-3

1. What are "equally likely outcomes"?
2. How does theoretical probability differ from empirical probability?
3. Can a theoretical probability ever exceed 1? Why or why not?
4. A certain baseball player has a batting average (number of hits/number of times at bat) of 0.375. What is the probability that he will not get a hit in his next at bat?

All of the letters in the word *Mississippi* are written on separate pieces of paper and put in a hat.

5. What is the probability of drawing the letter *s* from the hat?
6. What is the probability of drawing the letter *p*?
7. What is the probability of drawing the letter *a*?
8. What is the probability of drawing the letter *m* or *i*?

Each of the whole numbers 1 to 20 is written separately on 20 ping-pong balls and placed in a hat.

9. What is the probability that you will reach into the hat and draw out an even number?
10. What is the probability that you will draw a number that is evenly divisible by 5?
11. What is the probability that you will draw the number 3 on the first draw?
12. What is the probability that you will draw a number less than 5 on the first draw?

A card is drawn from a standard deck of playing cards.

13. What is the probability that the card is a 3?
14. What is the probability that the card is not a 3?
15. What is the probability that the card is a heart?
16. What is the probability that the card is a red card?
17. What is the probability that the card is a red spade?
18. What is the probability that the card is a black face card?
19. What is the probability that the card is a face card?
20. What is the probability that the card is a card with a value greater than 5 but less than 9?

21. What is the probability of drawing an ace?
22. What is the probability of drawing a red four?
23. What is the probability of drawing a queen or king of hearts?
24. What is the probability of drawing a card that is not a diamond?

A bag contains three red, four green, and five blue marbles. One marble is drawn from the bag.

25. What is the probability that the marble is red?
26. What is the probability that the marble is not blue?
27. What is the probability that the marble is red or green?
28. What is the probability that the marble is not red or green?

A fair, 6-sided die is tossed.

29. What is the probability of tossing a 5?
30. What is the probability of tossing a 4?
31. What is the probability of tossing an odd number?
32. What is the probability that the number shown is even?
33. What is the probability that the number shown is less than 3?
34. What is the probability that the number shown is greater than 6?
35. What is the probability of tossing a 1 or a 4?
36. What is the probability of tossing a number greater than 5 or an even number?
37. What is the probability of tossing an odd number or a 6?
38. What is the probability of tossing an odd number and a 6?

M&Ms® are packaged so that 24% in a given package are blue, 20% orange, 16% green, 14% yellow, 13% red, and 13% brown.

39. What is the probability that you will reach into a bag of these candies and remove a brown-colored candy?

40. What is the probability of pulling out a green M&M?
41. What is the probability of drawing a purple M&M?
42. What is the probability of drawing an M&M that is not green?

A TV remote has keys for channels 0–9. You select one key at random.

43. What is the probability that you select channel 6?
44. What is the probability that you select a channel that is an odd number?
45. What is the probability that you select a channel less than 8?
46. What is the probability that you don't select 1?

A multiple-choice test has six possible answers for each question.

47. If you guess at an answer, what is the probability that you select the correct answer for one particular question?
48. If you guess at an answer, what is the probability that you select the incorrect answer for one particular question?
49. If you correctly eliminate two of the six possible answers and guess from the remaining possibilities, what is the probability that you select the correct answer to that question?
50. If you correctly eliminate two of the six possible answers and guess from the remaining possibilities, what is the probability that you select the incorrect answer to the problem?

A traffic light is red for 35 sec, yellow for 10 sec, and green for 45 sec.

51. What is the probability that when you reach the light, the light will be red?

52. What is the probability that when you reach the light, the light will be yellow?
53. What is the probability that when you reach the light, the light will not be red?
54. What is the probability that when you reach the light, the light will not be red or yellow?

A bin contains 100 batteries (all size C). There are 40 Eveready, 24 Duracell, 20 Sony, 10 Panasonic, and 6 Rayovac batteries. One battery is selected at random from the bin.

55. Find the probability that the battery selected is a Duracell.
56. Find the probability that the battery selected is a Duracell or Eveready.
57. Find the probability that the battery selected is a Duracell, Eveready, or Sony.
58. Find the probability that the battery selected is a Kodak.
59. Toss a fair coin 10 times and record the number of times that it lands heads up. Based on this experiment, what is the empirical probability of tossing heads? How does this compare to the theoretical probability of tossing heads? Repeat the experiment by tossing a coin 50 times and recording the number of times that it lands heads up. Is the empirical probability in the second experiment closer to the theoretical probability?
60. Roll a fair die 20 times and record the number of times that the number 5 occurs. Based on this experiment, what is the empirical probability of rolling 5 on a die? How does this compare to the theoretical probability of rolling 5 on a die? Repeat the experiment by rolling a die 100 times and recording the number of times that the number 5 occurs. Is the empirical probability in the second experiment closer to the theoretical probability?

Section

7-4

Odds

The odds of an event occurring or not occurring are directly related to the probability of the events. *The odds in favor of* an event are expressed as the ratio of a pair of integers, which is the ratio of the probability that an event will happen to the probability that it will not happen. For example, there are six numbers on a die. The

Example

5

Calculating Probabilities from Odds

The National Safety Council lists the odds of dying from a lightning strike as 1 to 126,158. Use these odds to calculate the probability of dying from a lightning strike (Source: National Safety Council, *Injury Facts*®, 2013 Edition)

We know that probability is defined as

$$P(E) = \frac{\text{number of ways event } E \text{ can occur}}{\text{total number of possible outcomes}}$$

To calculate total number of outcomes of an experiment using odds, we add the two numbers used to create the ratio, 1 and 126,158.

$$\text{Total Outcomes} = 1 + 126,158 = 126,159$$

$$\text{Therefore, } P(\text{dying from a lightning strike}) = \frac{1}{126,159}$$

Practice Set 7-4

Calculate odds in favor of an event given the following sets of probabilities.

1. $P(\text{birth of a boy}) = \frac{1}{2}$ 1:1

2. $P(\text{rolling a 4 on a die}) = \frac{1}{6}$ 1:5

3. $P(\text{winning a game}) = \frac{2}{5}$ 2:3

4. $P(\text{guessing correct answer}) = \frac{1}{4}$

5. $P(\text{drawing an ace}) = \frac{1}{13}$

6. $P(\text{drawing a heart}) = \frac{1}{4}$

Calculate the probability given the odds in favor of each event.

7. Odds of rolling an even number on a die—1:1

8. Odds of rolling a 1 or 3 on a die—1:3

9. Odds of winning a race—3:11

10. Odds of guessing a correct answer out of 5 possibilities—1:4

11. Odds of drawing the number 10 from a deck of cards—1:12

12. Odds of drawing a face card from a deck—3:10

13. The odds in favor of winning the door prize are 3 to 17. Find the odds against winning the door prize.

14. The odds against Sparkles Pretty winning the horse race are 5:2. Find the odds in favor of Sparkles Pretty winning.

15. A committee of 3 is to be chosen from your math class members. If there are 20 members in your class, what are the odds that you will be chosen to be on this committee?

16. One person is selected at random from a class of 19 males and 13 females. Find the odds against selecting

(a) a female.

(b) a male.

You roll a fair, 6-sided die one time. Use this information to answer problems 17 to 22.

17. (a) Find the probability of rolling a 5.
(b) Find the odds of rolling a 5.

18. (a) Find the probability of not rolling a 5.
(b) Find the odds against rolling a 5.

19. Find the odds against rolling an odd number.

20. Find the odds against rolling a number greater than 4.

21. Find the odds in favor of rolling a number less than 3.
22. Find the odds in favor of rolling a number greater than 4.

Given a standard deck of 52 playing cards, you draw one card. Use this information for problems 23 to 28.

23. (a) Find $P(\text{club})$.
(b) Find the odds of drawing a club.
24. (a) Find $P(\text{ace})$.
(b) Find the odds of drawing an ace.
25. Find the odds in favor of selecting a 6.
26. Find the odds in favor of selecting a heart.
27. Find the odds against selecting a face card.
28. Find the odds against drawing an ace.
29. Suppose the probability that you are asked to work overtime this week is $\frac{5}{8}$. Find the odds in favor of your being asked to work overtime.
30. Suppose that the probability that a mechanic fixes your car right the first time is 0.7. Find the odds against your car being repaired right on the first attempt.
31. One million tickets are sold for a raffle. If you purchase one ticket, find your odds against winning (odds of losing).
32. One million tickets are sold for a raffle. If you purchase 10 tickets, find your odds against winning (odds of losing).
33. You bet on a horse in a race. If the odds on the horse are 5:1, what is the probability that the horse will win the race?

34. If the odds in a certain game are 5:9 in your favor, what is the probability that you will win the game?
35. The odds against Julie Wilson being admitted to the college of her choice are 9:2. Find the probability that Julie will be admitted.
36. The odds against Julie Wilson being admitted to the college of her choice are 9:2. Find the probability that Julie will not be admitted.
37. The odds against Jason getting promoted are 5:9. Find the probability that Jason will get promoted.
38. The odds against Jason getting promoted are 5:9. Find the probability that Jason will not get promoted.
39. The odds in favor of Kristen winning the spelling bee are 7:5. Find the probability that Kristen will win the spelling bee.
40. The odds in favor of Kristen winning the spelling bee are 7:5. Find the probability that Kristen will lose the spelling bee.
41. The odds in favor of Tim winning the tennis match are 1:6. Find the probability that Tim will win the match.
42. The odds in favor of Tim winning the tennis match are 1:6. Find the probability that Tim will not win the match.
43. The odds against Brian winning the 100-yard dash are 5:2. Find the probability that Brian will win.
44. The odds against Brian winning the 100-yard dash are 5:2. Find the probability that Brian will lose.

Exam

Exam

Exam

Section

7-5

Tree Diagrams

Even when situations seem simple, the number of possible results can be quite large. It is often important to know the number of possibilities that a situation may present before beginning to calculate the probability of any individual event.

As stated earlier, the possible results of an experiment are called outcomes. To solve more difficult probability problems, you must first be able to determine all the possible outcomes of the experiment. There are two methods that may be used to help you determine the number of outcomes. These methods are the **counting principle** and **tree diagrams**.

(b) See Figure 7-10. The sample space is $S = \{RP, RG, RB, PR, PG, PB, GR, GP, GB, BR, BP, BG\}$.

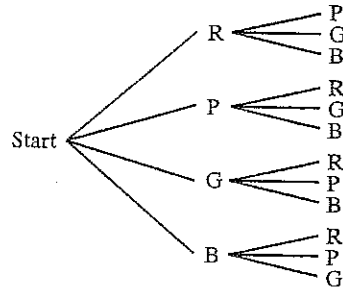


FIGURE 7-10
Tree diagram for drawing marbles.

(c) Recall that

$$P(E) = \frac{\text{number of ways event } E \text{ can occur}}{\text{total number of possible outcomes}}$$

The total number of outcomes is 12. Six outcomes have one purple marble: PR, PG, PB, RP, GP, or BP.

$$P(\text{one purple marble is selected}) = \frac{6}{12} = \frac{1}{2}$$

(d) One possible outcome meets the criteria for this problem: BG

$$P(\text{blue followed by green}) = \frac{1}{12}$$

Practice Set 7-5

Use the counting principle to determine the number of possible outcomes for each of the following.

- How many different four-digit numbers can be formed from the digits 0–9 if the first digit must be even and cannot be 0? (You may repeat digits.)
- How many different four-digit numbers can be formed from the digits 0–9 if the first digit must be even and cannot be 0? (You may not repeat digits.)
- A bag contains five different lightbulbs. The bulbs are the same size, but each has a different wattage. If you select three bulbs at random, how many different outcomes will be in the sample space if the bulbs are selected with replacement?
- A bag contains five different lightbulbs. The bulbs are the same size, but each has a different wattage. If you select three bulbs at random, how many different outcomes will be in the sample space if the bulbs are selected without replacement?
- Two marbles are to be selected from a bag without replacement. The bag contains one red, one blue, one green, one yellow, and one purple marble. How many possible outcomes will there be for this experiment?
- Two marbles are to be selected from a bag with replacement. The bag contains one red, one blue, one green, one yellow, and one purple marble. How many possible outcomes will there be for this experiment?













Draw a tree diagram for tossing a coin 3 times and then answer the following questions.

- What is the size of this sample space?
- What is the sample space for this experiment?

9. What is the probability that tossing a coin 3 times will result in one head and two tails?
10. What is the probability of tossing two or more heads?

A penny is tossed, and a fair, 6-sided die is rolled.

11. Use the counting principle to determine the number of outcomes in the sample space.
12. Construct a tree diagram illustrating all the possible outcomes and list the sample space.
13. What is the probability of tossing a head and rolling a 3?
14. What is the probability of tossing a head and rolling an even number?
15. Complete the following chart to show all of the possible outcomes for rolling two dice. Then answer the following questions.

						
	1, 1	1, 2				
						
						
						
						
						

- (a) What is the size of this sample space?
- (b) What is the probability of rolling a total of 7 on two dice?
- (c) What is the probability of rolling 13 on two dice?
- (d) What is the probability of rolling a 3 on both dice?

You perform an experiment that involves flipping a coin twice and then rolling a die once.

16. What is the size of the sample space?
17. What is the sample space?

18. Find $P(HH3)$.
19. Find $P(HT, \text{ in order, and then roll a } 5)$.
20. Find $P(TT \text{ and then roll an even number})$.

A bag contains three cards: an ace, a jack, and a queen. Two cards are to be selected at random with replacement.

21. Construct a tree diagram and determine the sample space.
22. Find the probability that two aces are selected.
23. Find the probability that a jack and then an ace are selected.
24. Find the probability that at least one queen is selected.

A couple plans to have two children.

25. Construct a tree diagram and list the sample space of the possible arrangements of boys and girls.
26. Find the probability that the family has two girls.
27. Find the probability that the family has at least one boy.
28. Find the probability that the family has at least two boys.

Suppose that you are going to make up a password for access to some files on your computer. You are going to choose three lowercase letters from the alphabet and then choose two digits (like *kuy76*).

29. How many different passwords are possible if repetition is allowed?
30. How many different passwords are possible if repetition is not allowed?
31. What is the probability that someone could guess your password on the first guess if the person knew to choose three letters and two digits?
32. What is the probability that someone could guess your password on the first guess if the person knew to choose three letters and two digits, and repetition is not allowed?

A monkey types the letters *o*, *k*, and *a* in random order. Answer the following questions.

33. How many different three-letter "words" could the monkey type using just these three letters? (You cannot repeat the letters.)
34. What is the probability that the monkey would type a meaningful word using these three letters?