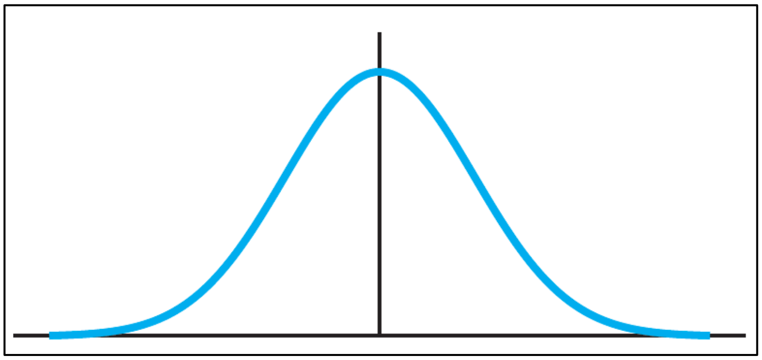
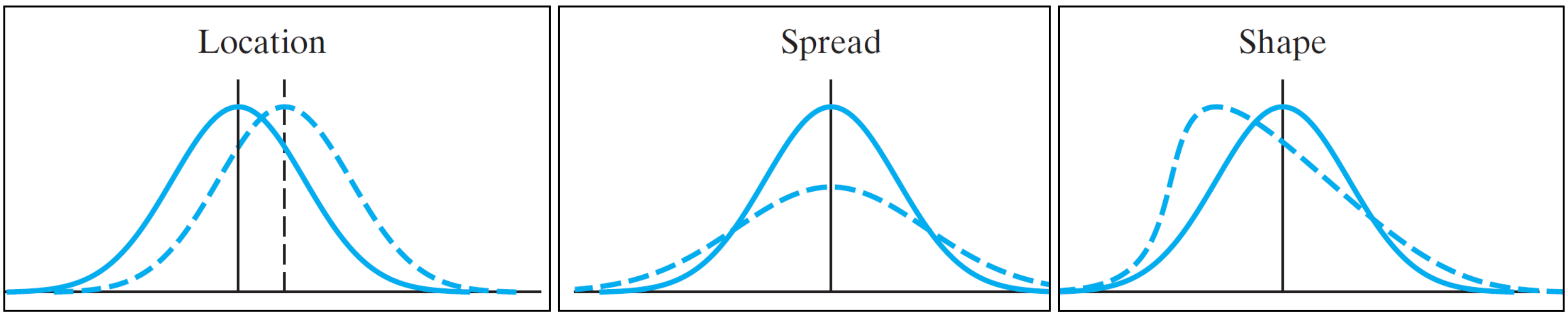
Normal Curve Notes

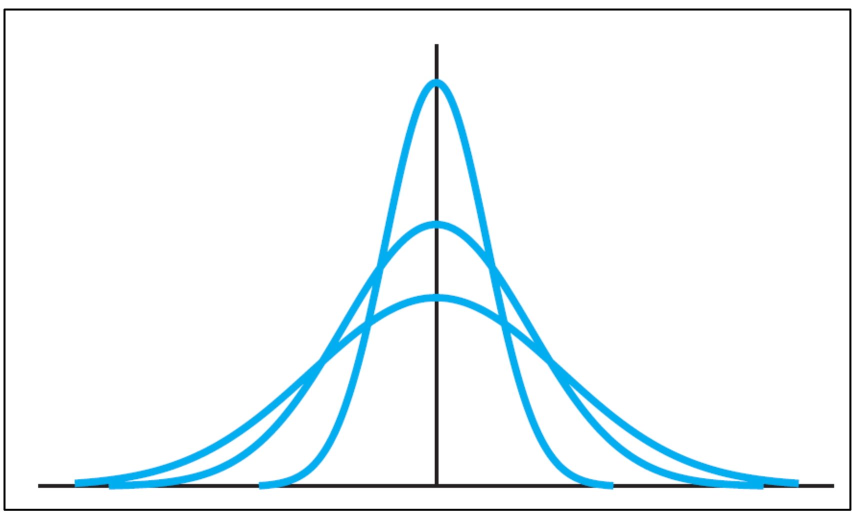
Many types of data will form a graph that is shaped like a normal curve (or bell curve).



Distributions of data that are collected can differ in many ways.



A series of normal curves with the same mean average but different standard deviations. (Example Below)



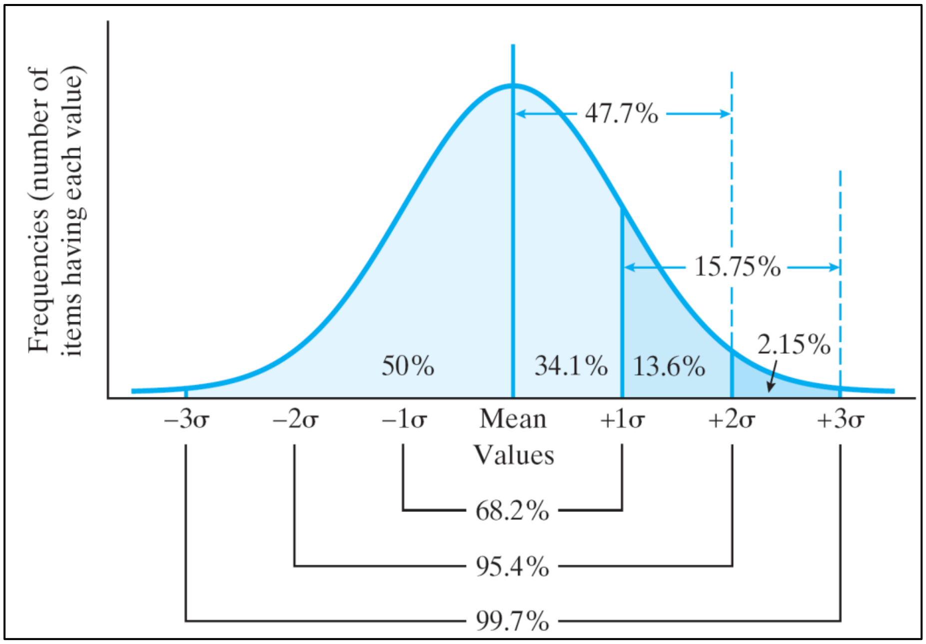
Once a set of data has been collected and is known to be approximately normally distributed, then the **empirical rule** can be applied to the analysis of the data

Theorem **The Empirical Rule**

For a distribution that is symmetrical and normally distributed:

1. Approximately 68.2% of all data values will lie within one standard deviation on either side of the mean.
2. Approximately 95.4% of all data values will lie within two standard deviations on either side of the mean.
3. Approximately 99.7% of all data values will lie within three standard deviations on either side of the mean.

Because the curve is **symmetrical**, these percentages will be split evenly on either side of the mean.



Definition **z-Score (Standard Score)**

A z-score is the number of standard deviations (s) that a particular piece of data (x) is from the mean average ($\overline{x}$) for the set of data. If a z-score is positive, then the piece of data in question is above the mean average, and if the z-score is negative, then it is below the mean average. A z-score is calculated as follows:

$$z = \frac{x - \overline{x}}{s}$$

(See Figure 8-16.)

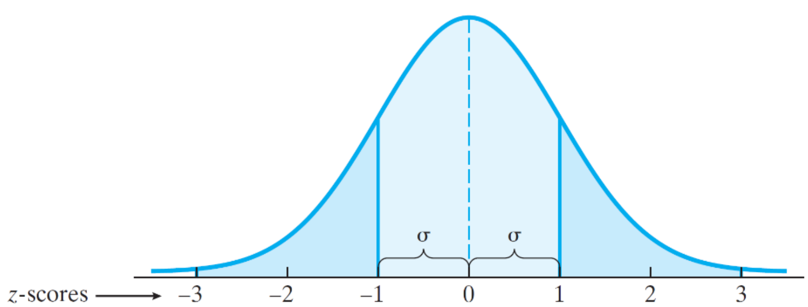


FIGURE 8-16

Normal curve and z-scores.

Example Problems:

1. Given a mean test score of 80 with a standard deviation of 5, calculate the following z-scores.

(a) What would your z-score be if you scored 85 on the test?

(b) What would your z-score be if you scored 70 on the test?

(c) What would your z-score be if you scored 93 on the test?

(d) What about for a test score of 90?

(e) If you had a z-score of −1.5 on this test, what numerical grade did you have?

1a.

$$z = \frac{x - \overline{x}}{s} \Longrightarrow = \frac{85 - 80}{5} = \frac{5}{5} = 1$$

1b. $z = \dfrac{x - \overline{x}}{s} \Longrightarrow = \dfrac{70 - 80}{5} = \dfrac{-10}{5} = -2$

1c. $z = \dfrac{x - \overline{x}}{s} \Longrightarrow = \dfrac{93 - 80}{5} = \dfrac{13}{5} = 2.6$, or 2.6 standard deviations above average

1d. $z = \dfrac{x - \overline{x}}{s} \Longrightarrow = \dfrac{90 - 80}{5} = \dfrac{10}{5} = 2$, or 2 standard deviations above average

1e. Here we will be solving the z-score formula for the value of x, the data value corresponding to your test grade.

$$z = \frac{x - \overline{x}}{s} \Longrightarrow -1.5 = \frac{x - 80}{5} = -7.5 = x - 80 \quad 72.5 = x$$